# Electrical Power Engineering

*By*

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# Lecture (3)

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# System Protection

In addition to generators, transformers, and transmission lines, other devices are required for the satisfactory operation and protection of a power system. Some of the protective devices directly connected to the circuits are called *switchgear*. They include instrument transformers, circuit breakers, disconnect switches, fuses and lightning arresters. These devices are necessary to deenergize either for normal operation or on the occurrence of faults. The associated control equipment and protective relays are placed on *switchboard* in *control houses*.

# Energy Control System

For reliable and economical operation of the power system it is necessary to monitor the entire system in a control center. The modern control center of today is called the *energy control center* (ECC). Energy control centers are equipped with on-line computers performing all signal processing through the remote acquisition system. Computers work in a hierarchical structure to properly coordinate different functional requirements in normal as well as emergency conditions. Every energy control center contains a control console which consists of a visual display unit (VDU), keyboard, and light pen. Computers may give alarms as advance warnings to the operators (dispatchers) when deviation from the normal state occurs. The dispatcher makes judgments and decisions and executes them with the aid of a computer. Simulation tools and software packages written in high-level language are implemented for efficient operation and reliable control of the system. This is referred to as SCADA, an acronym for "supervisory control and data acquisition."

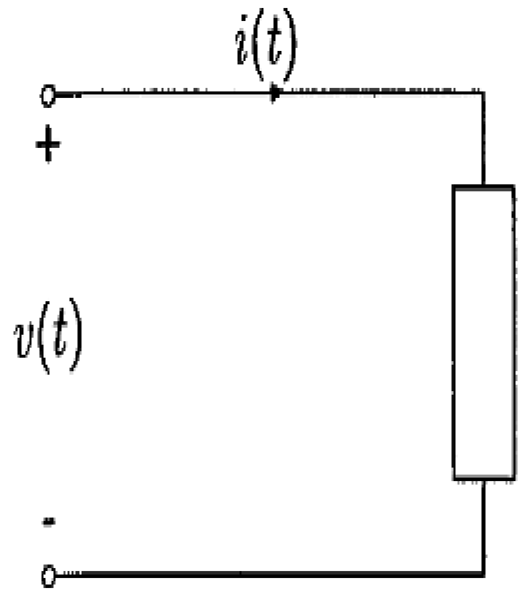
# Computer Analysis

For a power system to be practical it must be safe, reliable, and economical. Thus many analyses must be performed to design and operate an electrical system. However, before going into system analysis we have to model all components of electrical power systems. Therefore, in this text, after reviewing the concepts of power and three-phase circuits, we will calculate the parameters of a multi-circuit transmission line. Then, we will model the transmission line and look at the performance of the transmission line. Since transformers and generators are a part of the system, we will model these devices. Design of a power system, its operation and expansion requires much analysis. This text presents methods of power system analysis with the aid of a personal computer and the use of *MATLAB*.

# Basic Principles

# Power in Single Phase AC Circuits

Figure 2.1 shows a single-phase sinusoidal voltage supplying a load.




Let the instantaneous voltage be

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad (2.1)$$

and the instantaneous current be given by

$$i(t) = I_m \cos(\omega t + \theta_i) \qquad (2.2)$$

The instantaneous power $p(t)$ delivered to the load is the product of voltage $v(t)$ and current $i(t)$ given by

$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \qquad (2.3)$$

# Power in Single Phase AC Circuits

It is informative to write (2.3) in another form using the trigonometric identity

$$\cos A \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B) \qquad (2.4)$$

which results in

$$\begin{aligned} p(t) &= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \\ &= \frac{1}{2}V_m I_m\{\cos(\theta_v - \theta_i) + \cos[2(\omega t + \theta_v) - (\theta_v - \theta_i)]\} \\ &= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + \cos 2(\omega t + \theta_v)\cos(\theta_v - \theta_i) \\ &\quad + \sin 2(\omega t + \theta_v)\sin(\theta_v - \theta_i)] \end{aligned}$$

# Power in Single Phase AC Circuits

The *root-mean-square* (rms) value of $v(t)$ is $|V| = V_m/\sqrt{2}$ and the rms value of $i(t)$ is $|I| = I_m/\sqrt{2}$. Let $\theta = (\theta_v - \theta_i)$. The above equation, in terms of the rms values, is reduced to

$$p(t) = \underbrace{|V||I|\cos\theta[1 + \cos 2(\omega t + \theta_v)]}_{\substack{p_R(t) \\ \text{Energy flow into} \\ \text{the circuit}}} + \underbrace{|V||I|\sin\theta\sin 2(\omega t + \theta_v)}_{\substack{p_X(t) \\ \text{Energy borrowed and} \\ \text{returned by the circuit}}} \tag{2.5}$$

where $\theta$ is the angle between voltage and current, or the impedance angle. $\theta$ is positive if the load is inductive, (i.e., current is lagging the voltage) and $\theta$ is negative if the load is capacitive (i.e., current is leading the voltage).

# Power in Single Phase AC Circuits

The instantaneous power has been decomposed into two components. The first component of (2.5) is

$$p_R(t) = |V||I|\cos\theta + |V||I|\cos\theta\cos 2(\omega t + \theta_v)] \qquad (2.6)$$

The second term in (2.6), which has a frequency twice that of the source, accounts for the sinusoidal variation in the absorption of power by the resistive portion of the load. Since the average value of this sinusoidal function is zero, the average power delivered to the load is given by

$$P = |V||I|\cos\theta \qquad (2.7)$$

# Power in Single Phase AC Circuits

This is the power absorbed by the resistive component of the load and is also referred to as the *active power* or *real power*. The product of the rms voltage value and the rms current value $|V||I|$ is called the *apparent power* and is measured in units of volt ampere. The product of the apparent power and the cosine of the angle between voltage and current yields the real power. Because $\cos\theta$ plays a key role in the determination of the average power, it is called *power factor*. When the current lags the voltage, the power factor is considered lagging. When the current leads the voltage, the power factor is considered leading.

# Power in Single Phase AC Circuits

The second component of (2.5)

$$p_X(t) = |V||I| \sin\theta \sin 2(\omega t + \theta_v) \tag{2.8}$$

pulsates with twice the frequency and has an average value of zero. This component accounts for power oscillating into and out of the load because of its reactive element (inductive or capacitive). The amplitude of this pulsating power is called *reactive power* and is designated by $Q$.

$$Q = |V||I| \sin\theta \tag{2.9}$$

# Power in Single Phase AC Circuits

Both $P$ and $Q$ have the same dimension. However, in order to distinguish between the real and the reactive power, the term "var" is used for the reactive power (var is an acronym for the phrase "volt-ampere reactive"). For an inductive load, current is lagging the voltage, $\theta = (\theta_v - \theta_i) > 0$ and $Q$ is positive; whereas, for a capacitive load, current is leading the voltage, $\theta = (\theta_v - \theta_i) < 0$ and $Q$ is negative.

# Power in Single Phase AC Circuits

A careful study of Equations (2.6) and (2.8) reveals the following characteristics of the instantaneous power.

- For a pure resistor, the impedance angle is zero and the power factor is unity (UPF), so that the apparent and real power are equal. The electric energy is transformed into thermal energy.
- If the load is purely capacitive, the current leads the voltage by 90°, and the average power is zero, so there is no transformation of energy from electrical to nonelectrical form. In a purely capacitive circuit, the power oscillates between the source and the electric field associated with the capacitive elements.

# Power in Single Phase AC Circuits

- If the circuit is purely inductive, the current lags the voltage by 90° and the average power is zero. Therefore, in a purely inductive circuit, there is no transformation of energy from electrical to nonelectrical form. The instantaneous power at the terminal of a purely inductive circuit oscillates between the circuit and the source. When $p(t)$ is positive, energy is being stored in the magnetic field associated with the inductive elements, and when $p(t)$ is negative, energy is being extracted from the magnetic fields of the inductive elements.

# Complex Power

$$v(t) = V_m \cos(\omega t + \theta_v) \tag{2.1}$$

$$i(t) = I_m \cos(\omega t + \theta_i) \tag{2.2}$$

The rms voltage phasor of (2.1) and the rms current phasor of (2.2) shown in Figure 2.3 are

$$V = |V|\angle\theta_v \text{ and } I = |I|\angle\theta_i$$

The term $VI^*$ results in

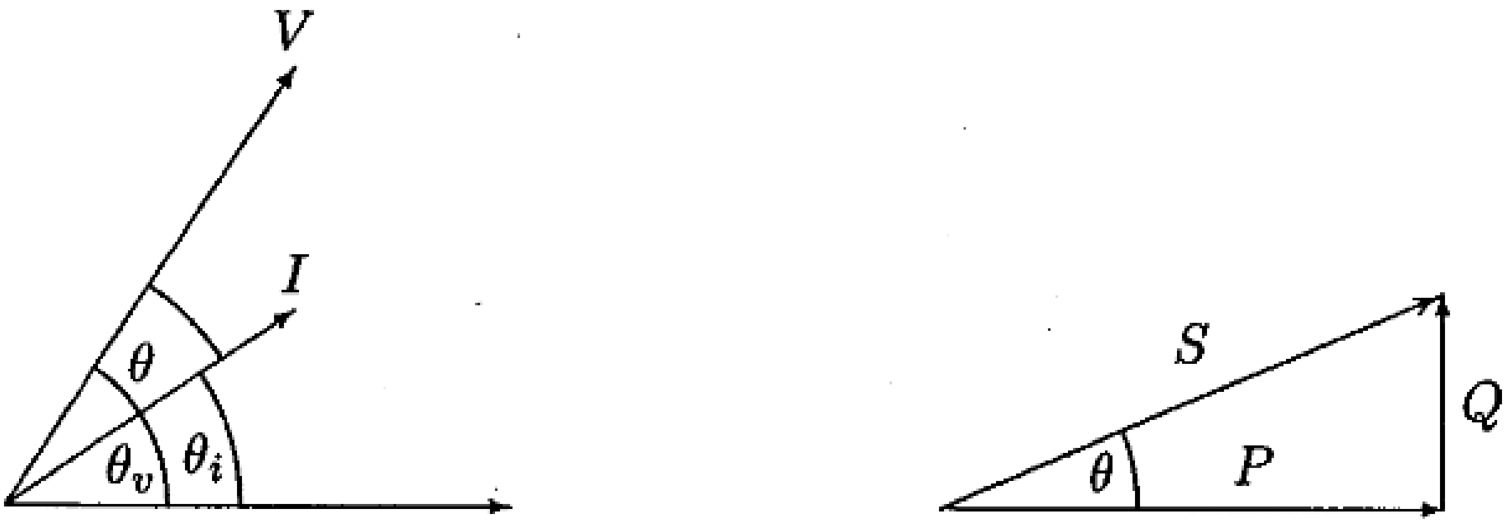



**FIGURE 2.3**
Phasor diagram and power triangle for an inductive load (lagging PF).

# Complex Power

$$VI^* = |V||I|\angle\theta_v - \theta_i = |V||I|\angle\theta$$

$$= |V||I|\cos\theta + j|V||I|\sin\theta$$

The above equation defines a complex quantity where its real part is the average (real) power $P$ and its imaginary part is the reactive power $Q$. Thus, the complex power designated by $S$ is given by

$$S = VI^* = P + jQ \tag{2.10}$$

The magnitude of $S$, $|S| = \sqrt{P^2 + Q^2}$, is the apparent power;

# Complex Power

The reactive power $Q$ is positive when the phase angle $\theta$ between voltage and current (impedance angle) is positive (i.e., when the load impedance is inductive, and $I$ lags $V$). $Q$ is negative when $\theta$ is negative (i.e., when the load impedance is capacitive and $I$ leads $V$) as shown in Figure 2.4.

In working with Equation (2.10 ) it is convenient to think of $P$, $Q$, and $S$ as forming the sides of a right triangle as shown in Figures 2.3 and 2.4.


**FIGURE 2.4**
Phasor diagram and power triangle for a capacitive load (leading PF).

# Complex Power

If the load impedance is $Z$ then

$$V = ZI \qquad (2.11)$$

substituting for $V$ into (2.10) yields

$$S = VI^* = ZII^* = R|I|^2 + jX|I|^2 \qquad (2.12)$$

From (2.12) it is evident that complex power $S$ and impedance $Z$ have the same angle. Because the power triangle and the impedance triangle are similar triangles, the impedance angle is sometimes called the *power angle*.

Similarly, substituting for $I$ from (2.11) into (2.10) yields

$$S = VI^* = \frac{VV^*}{Z^*} = \frac{|V|^2}{Z^*} \qquad (2.13)$$

From (2.13), the impedance of the complex power $S$ is given by

$$Z = \frac{|V|^2}{S^*} \qquad (2.14)$$

# The Complex Power Balance

From the conservation of energy, it is clear that real power supplied by the source is equal to the sum of real powers absorbed by the load. At the same time, a balance between the reactive power must be maintained. Thus the total complex power delivered to the loads in parallel is the sum of the complex powers delivered to each. Proof of this is as follows:

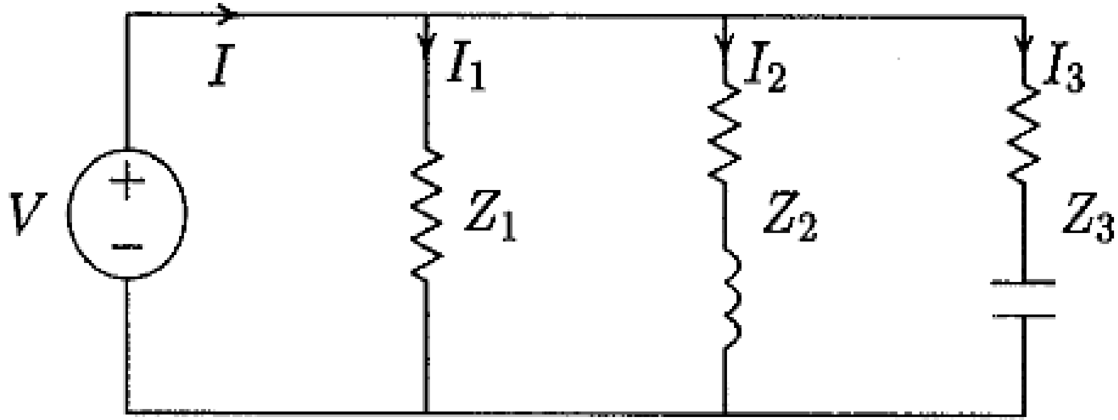



**FIGURE 2.5**
Three loads in parallel.

For the three loads shown in Figure 2.5, the total complex power is given by

$$S = VI^* = V[I_1 + I_2 + I_3]^* = VI_1^* + VI_2^* + VI_3^* \qquad (2.15)$$

# Complex Power

**Example 2.2**

In the above circuit $V = 1200\angle 0°$ V, $Z_1 = 60 + j0\ \Omega$, $Z_2 = 6 + j12\ \Omega$ and $Z_3 = 30 - j30\ \Omega$. Find the power absorbed by each load and the total complex power.

$$I_1 = \frac{1200\angle 0°}{60\angle 0} = 20 + j0 \text{ A}$$

$$I_2 = \frac{1200\angle 0°}{6 + j12} = 40 - j80 \text{ A}$$

$$I_3 = \frac{1200\angle 0°}{30 - j30} = 20 + j20 \text{ A}$$

$$S_1 = VI_1^* = 1200\angle 0°(20 - j0) = 24,000 \text{ W} + j0 \text{ var}$$

$$S_2 = VI_2^* = 1200\angle 0°(40 + j80) = 48,000 \text{ W} + j96,000 \text{ var}$$

$$S_3 = VI_3^* = 1200\angle 0°(20 - j20) = 24,000 \text{ W} - j24,000 \text{ var}$$

# Complex Power

The total load complex power adds up to

$$S = S_1 + S_2 + S_3 = 96,000 \text{ W} + j72,000 \text{ var}$$

Alternatively, the sum of complex power delivered to the load can be obtained by first finding the total current.

$$I = I_1 + I_2 + I_3 = (20 + j0) + (40 - j80) + (20 + j20)$$
$$= 80 - j60 = 100\angle -36.87^\circ \text{ A}$$

and

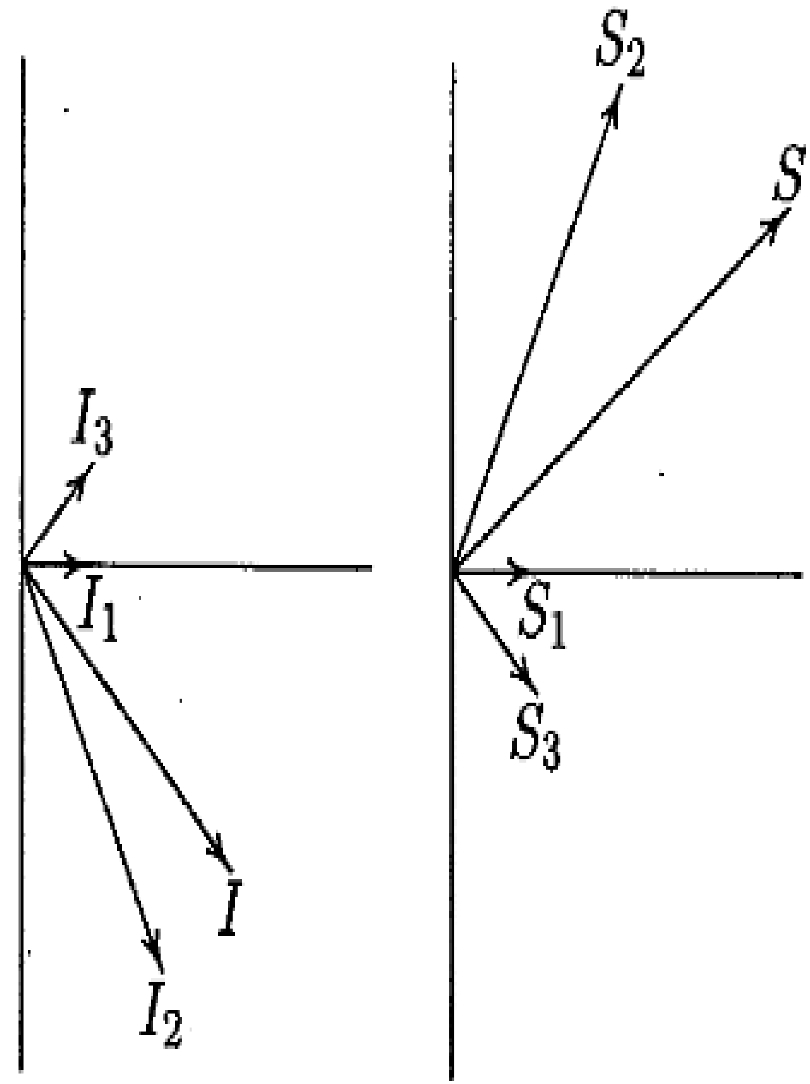
$$S = VI^* = (1200\angle 0^\circ)(100\angle 36.87^\circ) = 120,000\angle 36.87^\circ \text{ VA}$$
$$= 96,000 \text{ W} + j72,000 \text{ var}$$

A final insight is contained in Figure 2.6, which shows the current phasor diagram and the complex power vector representation.

# Complex Power

**FIGURE 2.6**

Current phasor diagram and power plane diagram.



# Complex Power

The complex powers may also be obtained directly from (2.14)

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(1200)^2}{60} = 24,000 \text{ W} + j\,0$$

$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(1200)^2}{6 - j12} = 48,000 \text{ W} + j96,000 \text{ var}$$

$$S_3 = \frac{|V|^2}{Z_3^*} = \frac{(1200)^2}{30 + j30} = 24,000 \text{ W} - j24,000 \text{ var}$$

# Power Factor Correction

$$P = |V||I|\cos\theta \qquad (2.7)$$

It can be seen from (2.7) that the apparent power will be larger than $P$ if the power factor is less than 1. Thus the current $I$ that must be supplied will be larger for $PF < 1$ than it would be for $PF = 1$, even though the average power $P$ supplied is the same in either case. A larger current cannot be supplied without additional cost to the utility company. Thus, it is in the power company's (and its customer's) best interest that major loads on the system have power factors as close to 1 as possible. In order to maintain the power factor close to unity, power companies install banks of capacitors throughout the network as needed. They also impose an additional charge to industrial consumers who operate at low power factors. Since industrial loads are inductive and have low lagging power factors, it is beneficial to install capacitors to improve the power factor. This consideration is not important for residential and small commercial customers because their power factors are close to unity.

# Power Factor Correction

**Example 2.3**

Two loads $Z_1 = 100 + j0\ \Omega$ and $Z_2 = 10 + j20\ \Omega$ are connected across a 200-V rms, 60-Hz source as shown in Figure 2.7.

(a) Find the total real and reactive power, the power factor at the source, and the total current.

(b) Find the capacitance of the capacitor connected across the loads to improve the overall power factor to 0.8 lagging.

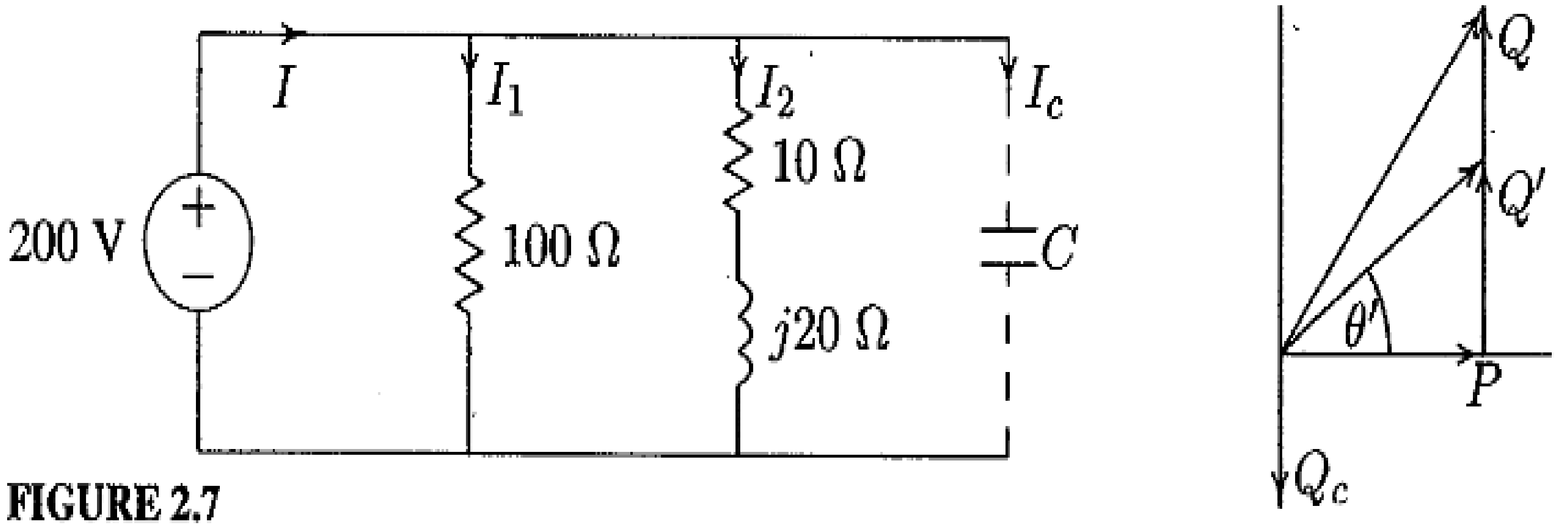



**FIGURE 2.7**

Circuit for Example 2.3 and the power triangle.

# Power Factor Correction

(a)

$$I_1 = \frac{200\angle 0^\circ}{100} = 2\angle 0^\circ \text{ A}$$

$$I_2 = \frac{200\angle 0^\circ}{10 + j20} = 4 - j8 \text{ A}$$

$$S_1 = VI_1^* = 200\angle 0^\circ(2 - j0) = 400 \text{ W} + j0 \text{ var}$$

$$S_2 = VI_2^* = 200\angle 0^\circ(4 + j8) = 800 \text{ W} + j1600 \text{ var}$$

Total apparent power and current are

$$S = P + jQ = 1200 + j1600 = 2000\angle 53.13^\circ \text{ VA}$$

$$I = \frac{S^*}{V^*} = \frac{2000\angle -53.13^\circ}{200\angle 0^\circ} = 10\angle -53.13^\circ \text{ A}$$

Power factor at the source is

$$PF = \cos(53.13) = 0.6 \text{ lagging}$$

# Power Factor Correction

(b) Total real power $P = 1200$ W at the new power factor 0.8 lagging. Therefore

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$
$$Q' = P \tan\theta' = 1200 \tan(36.87^\circ) = 900 \text{ var}$$
$$Q_c = 1600 - 900 = 700 \text{ var}$$
$$Z_c = \frac{|V|^2}{S_c^*} = \frac{(200)^2}{j700} = -j57.14\ \Omega$$
$$C = \frac{10^6}{2\pi(60)(57.14)} = 46.42\ \mu\text{F}$$

The total power and the new current are

$$S' = 1200 + j900 = 1500\angle 36.87^\circ$$
$$I' = \frac{S'^*}{V^*} = \frac{1500\angle -36.87^\circ}{200\angle 0^\circ} = 7.5\angle -36.87^\circ$$

Note the reduction in the supply current from 10 A to 7.5 A.

# Power Factor Correction

**Example 2.4**

Three loads are connected in parallel across a 1400-V rms, 60-Hz single-phase supply as shown in Figure 2.8.

Load 1: Inductive load, 125 kVA at 0.28 power factor.

Load 2: Capacitive load, 10 kW and 40 kvar.

Load 3: Resistive load of 15 kW.

(a) Find the total kW, kvar, kVA, and the supply power factor.

(b) A capacitor of negligible resistance is connected in parallel with the above loads to improve the power factor to 0.8 lagging. Determine the kvar rating of this capacitor and the capacitance in $\mu$F.

# Power Factor Correction

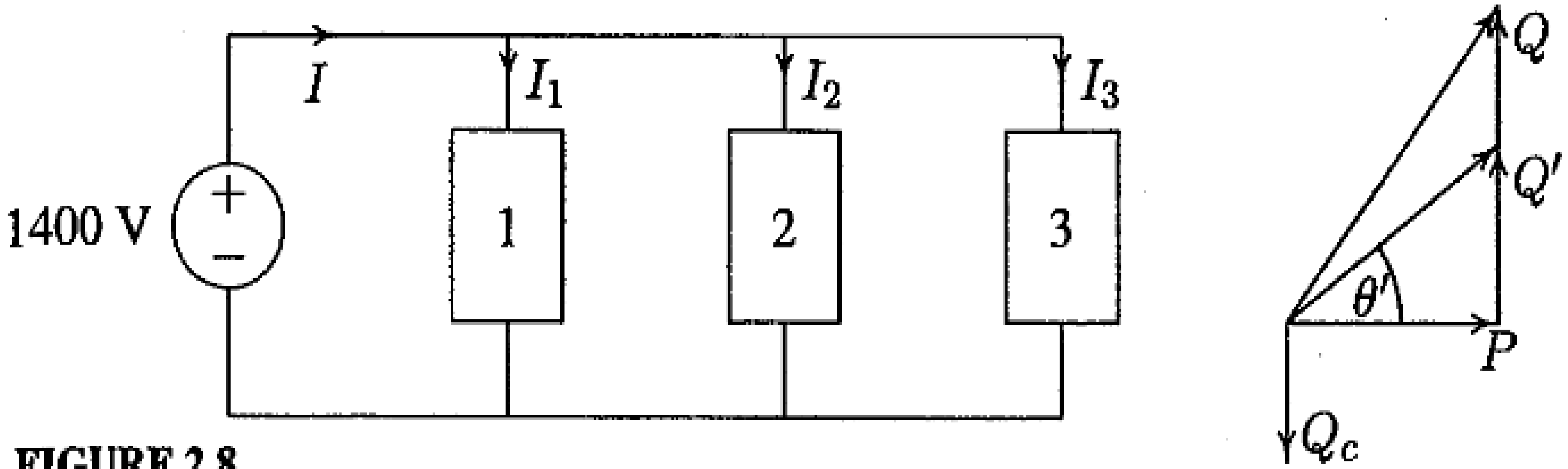



**FIGURE 2.8**
Circuit for Example 2.4.

# Power Factor Correction

(a) An inductive load has a lagging power factor, the capacitive load has a leading power factor, and the resistive load has a unity power factor.

For Load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74^\circ$ lagging

The load complex powers are

$$S_1 = 125\angle 73.74 \text{ kVA} = 35 \text{ kW} + j120 \text{ kvar}$$

$$S_2 = 10 \text{ kW} - j40 \text{ kvar}$$

$$S_3 = 15 \text{ kW} + j0 \text{ kvar}$$

The total apparent power is

$$\begin{aligned} S = P + jQ &= S_1 + S_2 + S_3 \\ &= (35 + j120) + (10 - j40) + (15 + j0) \\ &= 60 \text{ kW} + j80 \text{ kvar} = 100\angle 53.13 \text{ kVA} \end{aligned}$$

# Power Factor Correction

The total current is

$$I = \frac{S^*}{V^*} = \frac{100,000\angle -53.13^\circ}{1400\angle 0^\circ} = 71.43\angle -53.13^\circ \text{ A}$$

The supply power factor is

$$PF = \cos(53.13) = 0.6 \text{ lagging}$$

(b) Total real power $P = 60$ kW at the new power factor of 0.8 lagging results in the new reactive power $Q'$.

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q' = 60\tan(36.87^\circ) = 45 \text{ kvar}$$

# Power Factor Correction

Therefore, the required capacitor kvar is

$$Q_c = 80 - 45 = 35 \text{ kvar}$$

and

$$X_c = \frac{|V|^2}{S_c^*} = \frac{1400^2}{j35,000} = -j56\ \Omega$$

$$C = \frac{10^6}{2\pi(60)(56)} = 47.37\ \mu\text{F}$$

and the new current is

$$I' = \frac{S'^*}{V^*} = \frac{60,000 - j45,000}{1400\angle 0^\circ} = 53.57\angle -36.87^\circ \text{ A}$$

Note the reduction in the supply current from 71.43 A to 53.57 A.